ANNUAL PROGRESS REPORT TO THE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

N.A.S.A. Research Grant NsG-544

Louis R. Bragg, Case Institute of Technology

November 1, 1965

1. Introduction. The research grant referred to in this report was awarded on November 1, 1963 and was renewed on November 1, 1964. The objective of this grant was to carry out studies in the combined areas of Special Functions and Solution Representations of Partial Differential Equations. At the present time, seven projects covered by this grant have been completed. A number of other projects are well under way that depend upon the results in the completed pieces of research. Two of the completed reports have served as Doctoral dissertations for two Ph.D. candidates in Mathematics at Case Institute of Technology. In the following, we give a list of completed items indicating the author, title, technical report status, and publication plans or status.

(i) L. R. Bragg; A Rodrigue's Formula for the Generalized Laguerre Polynomials; Submitted to N.A.S.A. as a manuscript but not as an official report; to appear in the American Mathematical Monthly.

(ii) L. R. Bragg; The Radial Heat Polynomials and Related Functions; Completed as an N.A.S.A. Technical Report in September 1964; To appear in the Transactions of the American Mathematical Society.

(iii) W. J. Davis; Solution Representations for Linear Initial Value Problems; Completed as an N.A.S.A. Technical Report in July, 1964 (also served as a Ph.D. Thesis); Submitted to a Roumanian Mathematics Journal.

(iv) A. T. Hopper; Expansion Theorems for Two Generalizations of the Heat Equation; Results were accepted as a Ph.D. Thesis and a completed report was submitted to N.A.S.A. in June, 1965. The publication plans are not complete since some of the work in that report is being extended by the author.

(v) L. R. Bragg; The Radial Heat Equation and Laplace Transforms; Completed as an N.A.S.A. Technical Report in August, 1965; a rewritten version will be submitted to, most likely, the Siam Reviews.

(vi) L. R. Bragg; On Associated Radial Heat Expansions; A report submitted with this progress report; Submitted to the Proceedings of the American Mathematical Society.

(vii) L. R. Bragg; A Definition of the Laplace Transform; This also accompanies this progress report: Submitted to the American Mathematical Monthly.

The remainder of this progress report will indicate the status of research in progress, intended direction of further research, and expenditures made from the grant.

2. Research in Progress. Listed below are a number of research projects that are at various stages of completion. The statements indicate the problem being treated, its significance to the project, and its nearness to completion.

(a) Solution Representation of the Radial Heat Equation Near Singular Points. The solutions of the equation

$$(*) \quad \frac{\partial u(r,t)}{\partial t} = \frac{\partial^2 u(r,t)}{\partial r^2} + \frac{u-1}{r} \frac{\partial u(r,t)}{\partial r}$$

are being examined in a neighborhood of $r = 0$ at which point certain solutions can exihibit singularities. The possibility of singularities in initial data is also being considered. This research will make use of item (v) listed in the introduction. The project is close to completion.

(b) Solutions of the Radial Heat Equation Corresponding to Odd Date Functions. The results announced in item (ii) in the introduction gave expansion theorems for solutions of (*) whenever the initial data was a function of r^2, that is, an even function of r. The results on Laplace Transforms now permits one to examine the solution structure corresponding to initial data that involves odd functions of r. The work on this project has just gotten under way.

(c) Distributions and the Radial Heat Equation. The connections between the Laplace transform and the heat equation (*) permit one to examine generalized solutions of this equation. Some results in this direction were announced in item (v) of the introduction. The direction in which this research will go is not entirely clear at this time. It appears as if a fair amount of work involving functional analysis will be needed. The treatment appears to be promising, however.

(d) One of my thesis students has started research on a problem related to the radial heat equation (*) in which a time dependent source of heat is given at $r = 0$. The problem is to determine the heat distribution $u(r,t)$ for any $r > 0$ and $t > 0$. Sets of polynomials analogous to the radial heat polynomials are being sought to treat expansion theorems for this problem. Successful work in this area would extend results of Widder and Powell and Poritsky.

3. Planned Research. At present, no major deviations from the originally proposed research is being planned. The results already obtained have led to problems not anticipated when the proposal was originally written. However, these newer topics fit into the general context of that proposal. The items (b), (c), and (d) noted in the section on Research in Progress will receive the major attention over the next six months period.

4. Expenditures. A detailed account of funds spent on this grant is being sent to N.A.S.A. by the business office of Case Institute of Technology.

lc